BIBLIOGRAPHY

1. Srivastav, R. P., Dual series relations, II. Dual relations involving Dini series. Proc. Roy. Soc., Ser. A, Vol. 66, pt. 3, 1962-1963.
2. Irwin, G. R., Fracture. Handbuch der Physik, Bd. 6, Springer-Verlag, Berlin, 1958.
3. Neuber, H., Stress Concentrations. Gostekhizdat, Moscow-Leningrad, 1947.
4. Sneddon, I. N. and Berry, D. S., Classical Elasticity Theory (Russian translation), Fizmatgiz, Moscow, 1961.
5. Sneddon, I. N. and Tait, R. J., The effect of a penny-shaped crack on the distribution of stress in a long circular cylinder. Internat. J. Engng. Sci., Vol. 1, №3, 1963.
6. Applied Questions of the Viscosity of Fracture. "Mir", Moscow, 1968.

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A SYSTEM OF ARBITRARILY ORIENTED CRACKS IN ELASTIC SOLIDS

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The plane problem of the theory of elasticity for an unbounded domain, containing N arbitrarily situated rectilinear cuts (cracks), is reduced to a system of N singular integral equations relative to functions which characterize the discontinuity of the displacements along the crack lines. The general solution of the integral equations for the case of distantly located cracks in the form of a power series with respect to a small parameter, is obtained. The problem of rupture is also considered.

In the plane theory of cracks there exist a series of investigations devoted to the study of the interactions between cracks which are ordered in a definite manner (colinear [1−3], parallel [4, 5], with a chessboard distribution [6]). By the representation of the complex potentials in the form of Laurent series [7], we determine approximately the state of stress of an unbounded plate, weakened by a system of arbitrarily oriented cracks, in the case of a linear distribution of stresses at infinity. We reduce the plane problem of the theory of elasticity for an infinite body, containing arbitrarily situated rectilinear cracks and with an arbitrary load, to a system of integral equations; this will allow to solve a series of new problems in the mathematical theory of cracks.

1. Assume that in an elastic plane, related to a Cartesian system of coordinates xOy, there exist N cuts (cracks) of length $2a_k$ $(k = 1, 2, \ldots, N)$. The centers O_k of the cracks are determined by the coordinates $z_{k0} = x_{k0} + iy_{k0} = d_k e^{i\beta_k}$. At the points O_k there are located the origins of local systems of coordinates $x_kO_ky_k$. The axes O_kx_k coincide with the crack lines and form the angles α_k with the axis Ox (Fig. 1). The boundaries of the cracks are loaded by the self-balancing forces

$$p_k(x_k) = N_k^+ - iT_k^+ = N_k^- - iT_k^-, \qquad |x_k| \leqslant a_k \qquad (k = 1, 2, \ldots, N) \quad (1.1)$$

The determination of the state of stress and strain in an infinite plane containing one crack $|x_k| \leqslant a_k$, $y_k = 0$, reduces to solving the singular integral equation [8, 9]

$$\int_{-a_k}^{a_k} \frac{g_k'(t)\,dt}{t - x_k} = \pi p_k(x_k), \qquad |x_k| \leqslant a_k$$

relative to the function which characterizes the discontinuity of the displacements

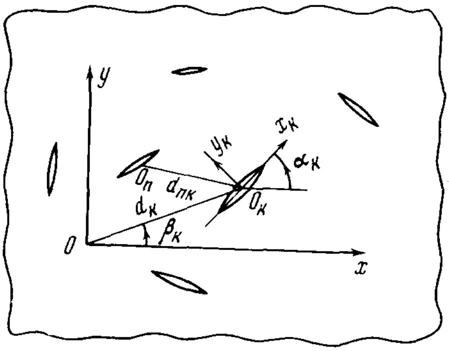

Fig. 1

$u(x, 0)$, $v(x, 0)$ at the line of the crack (G, $\varkappa$ are the elastic constants of the material)

$$g_k(x_k) = -\frac{2Gi}{1+\varkappa}\{i[v_k^+(x_k) - v_k^-(x_k)] + [u_k^+(x_k) - u_k^-(x_k)]\}$$

In terms of the known function $g_k'(x)$ we determine [10] the combination of the stresses $N_{nk} - iT_{nk}$ at the line $O_n x_n$

$$N_{nk} - iT_{nk} = \frac{1}{\pi}\int_{-a_k}^{a_k} [g_k'(t) K_{nk}(t, x_n) + \overline{g_k'(t)} L_{nk}(t, x_n)]\,dt$$

$$(n = 1, 2, \ldots, N, \quad n \neq k)$$

$$K_{nk}(t, x) = \overline{S_{nk}(t, x)} + S_{nk}(t, x)\, e^{2i(\alpha_k - \alpha_n)}$$

$$L_{nk}(t, x) = S_{nk}(t, x) - \frac{S_{nk}^2(t, x)}{\overline{S_{nk}(t, x)}}\, e^{2i(\alpha_k - \alpha_n)}$$

$$S_{nk}(t, x) = {}^1/_2\,[t - xe^{i(\alpha_k - \alpha_n)} - d_{nk}e^{i(\alpha_k - \beta_{nk})}]^{-1}$$

$$d_{nk}e^{i\beta_{nk}} = d_n e^{i\beta_n} - d_k e^{i\beta_k}$$

Considering the expression

$$-\sum_{k=1}^{N}{}' (N_{nk} - iT_{nk})$$

as an additional exterior load, applied to the boundary of the nth crack $|x_n| \leqslant a_n$, $y_n = 0$, we obtain a system of N singular integral equations relative to the unknown functions $g_n'(x_n)$

$$\int_{-a_n}^{a_n} \frac{g_n'(t)\,dt}{t-x} + \sum_{k=1}^{N}{}' \int_{-a_k}^{a_k} [g_k'(t)\,K_{nk}(t, x) + \overline{g_k'(t)}\,L_{nk}(t, x)]\,dt = \pi p_n(x)$$

$$|x| \leqslant a_n \quad (n = 1, 2, \ldots, N) \qquad (1.2)$$

The symbol Σ' means that at the summation the term corresponding to the row index is excluded. Making use of the formula for the conversion of Cauchy type integrals [11], we arrive at a system of Fredholm integral equations of the second kind

$$g_n'(x) = \frac{1}{\pi\sqrt{a_n^2 - x^2}} \left\{ -\int_{-a_n}^{a_n} \frac{\sqrt{a_n^2 - t^2}}{t-x} p_n(t)\,dt + \sum_{k=1}^{N}{}' \int_{-a_k}^{a_k} [g_k'(t)\,M_{nk}(t, x) + \overline{g_k'(t)}\,R_{nk}(t, x)]\,dt \right\}, \quad |x| \leqslant a_n \qquad (1.3)$$

$$(n = 1, 2, \ldots, N)$$

Here

$$M_{nk}(t, x) = \frac{1}{\pi} \int_{-a_n}^{a_n} \frac{\sqrt{a_n^2 - \xi^2}}{\xi - x} K_{nk}(t, \xi)\,d\xi,$$

$$R_{nk}(t, x) = \frac{1}{\pi} \int_{-a_n}^{a_n} \frac{\sqrt{a_n^2 - \xi^2}}{\xi - x} L_{nk}(t, \xi)\,d\xi$$

Thus, the plane problem of the theory of elasticity for an unbounded body with arbitrarily located rectilinear cracks is reduced to the system of singular integral equations (1.2), which, in turn, is transformed into the system of Fredholm integral equations of the second kind (1.3). We note that the kernels $M_{nk}(t, x)$, $R_{nk}(t, x)$ of the system (1.3) can be evaluated in closed form.

The system (1.2) or (1.3) allows to consider very different cases of distribution of cracks. In particular, we can obtain the integral equations of the periodic problem in the theory of cracks. If we make the length of one of the cracks tend to infinity, then we obtain the description of the state of stress of a semiplane, weakened by a system of arbitrarily located cracks. In what follows, we will find the general solution of the system (1.3) in the case when the cracks are situated at a large distance from each other. For closely situated or intersecting cracks, Eqs. (1.3) can be solved numerically.

2. For large distances between the centers of the cracks, the kernels $M_{nk}(t, x)$, and $R_{nk}(t, x)$ have the expansions

$$\begin{pmatrix} M_{nk}(t, x) \\ R_{nk}(t, x) \end{pmatrix} = \sum_{p=0}^{\infty} \sum_{\nu=0}^{p} \begin{pmatrix} a_{nkp\nu} \\ b_{nkp\nu} \end{pmatrix} H_{p-\nu}\left(\frac{x}{a_n}\right)\left(\frac{t}{a_n}\right)^{\nu} \left(\frac{\varepsilon_{kn}}{2}\lambda\right)^{p+1} \qquad (2.1)$$

Here

$$\lambda = \frac{2a}{d} < 1, \quad a = \max\{a_n\}, \quad d = \min\{d_{nk}\}, \quad \varepsilon_{nk} = \frac{a_k d}{a d_{nk}} \leqslant 1$$

$$H_p\left(\frac{x}{a_n}\right) = \frac{1}{\pi a_n^{p+1}} \int_{-a_n}^{a_n} \frac{\xi^p \sqrt{a_n^2 - \xi^2}}{\xi - x}\,d\xi \qquad (2.2)$$

$a_{nkp\nu}$, $b_{nkp\nu}$ are constant coefficients

$$
\begin{aligned}
a_{nk11} &= -\tfrac{1}{2}\left[e^{-2i(\beta_{nk}-\alpha_k)} + e^{2i(\beta_{nk}-\alpha_n)}\right],\\
a_{nk22} &= -\tfrac{1}{2}\left[e^{-3i(\beta_{nk}-\alpha_k)} + e^{i(3\beta_{nk}-2\alpha_n-\alpha_k)}\right]\\
b_{nk11} &= -\tfrac{1}{2}\left[e^{2i(\beta_{nk}-\alpha_k)} + e^{2i(\beta_{nk}-\alpha_n)} - 2e^{2i(2\beta_{nk}-\alpha_k-\alpha_n)}\right]\\
b_{nk22} &= -\tfrac{1}{2}\left[e^{3i(\beta_{nk}-\alpha_k)} + 2e^{i(3\beta_{nk}-\alpha_k-2\alpha_n)} - 3e^{i(5\beta_{nk}-3\alpha_k-2\alpha_n)}\right]\\
a_{nk21} &= e^{-i(3\beta_{nk}-2\alpha_k-\alpha_n)} + e^{3i(\beta_{nk}-\alpha_n)},\\
a_{nk33} &= -\tfrac{1}{2}\left[e^{-4i(\beta_{nk}-\alpha_k)} + e^{2i(2\beta_{nk}-\alpha_n-\alpha_k)}\right]\\
b_{nk21} &= 2e^{i(3\beta_{nk}-2\alpha_k-\alpha_n)} - 3e^{i(5\beta_{nk}-2\alpha_k-3\alpha_n)} + e^{3i(\beta_{nk}-\alpha_n)}\\
b_{nk33} &= -\tfrac{1}{2}\left[e^{4i(\beta_{nk}-\alpha_k)} + 3e^{2i(2\beta_{nk}-\alpha_k-\alpha_n)} - 4e^{2i(3\beta_{nk}-2\alpha_k-\alpha_n)}\right]\\
a_{nk32} &= \tfrac{3}{2}\left[e^{-i(4\beta_{nk}-3\alpha_k-\alpha_n)} + e^{i(4\beta_{nk}-3\alpha_n-\alpha_k)}\right]\\
b_{nk32} &= 3\left[e^{i(4\beta_{nk}-3\alpha_k-\alpha_n)} - 2e^{3i(2\beta_{nk}-\alpha_n-\alpha_k)} + e^{i(4\beta_{nk}-\alpha_k-3\alpha_n)}\right]\\
a_{nk31} &= -\tfrac{3}{2}\left[e^{4i(\beta_{nk}-\alpha_n)} + e^{-2i(2\beta_{nk}-\alpha_n-\alpha_k)}\right]\\
b_{nk31} &= -\tfrac{3}{2}\left[3e^{2i(2\beta_{nk}-\alpha_n-\alpha_k)} - 4e^{2i(3\beta_{nk}-2\alpha_n-\alpha_k)} + e^{4i(\beta_{nk}-\alpha_n)}\right] \quad \text{etc.}
\end{aligned}
\tag{2.3}
$$

We will seek the solution of the system (1.3) in the form of the series

$$g_n'(x) = \sum_{p=0}^{\infty} g_{np}'(x)\lambda^p \tag{2.4}$$

Inserting (2.1) and (2.4) into (1.3) and making equal the coefficients of the same powers of λ, we obtain a system of equations for the determination of $g_{np}'(x)$

$$g_{n0}'(x) = -\frac{1}{\pi\sqrt{a_n^2-x^2}}\int_{-a_n}^{a_n}\frac{\sqrt{a_n^2-t^2}\,p_n(t)}{t-x}\,dt, \quad g_{n1}'(x) = 0 \tag{2.5}$$

$$g_{np}'(x) = \frac{1}{\pi\sqrt{a_n^2-x^2}}\sum_{k=1}^{N}{}' \sum_{s=1}^{p-1}\sum_{\nu=1}^{s} H_{s-\nu}\left(\frac{x}{a_n}\right)\times$$

$$a_n^{-\nu}\left(\frac{\varepsilon_{kn}}{2}\right)^{s+1}\int_{-a_k}^{a_k} t^{\nu}\left[a_{nks\nu}g_{k,p-s-1}'(t) + b_{nks\nu}\overline{g_{k,p-s-1}'(t)}\right]dt \qquad (p=2,\ 3,\ \ldots)$$

Knowing the functions $g_n'(x)$, we can determine the state of stress and strain of the plane with arbitrarily oriented cuts. However, the obtained solution (2.4) is effective only for small values of the parameter λ, i.e. in the case when the cracks are situated at a large distance from each other.

3. We consider the problem of the limiting equilibrium [12] of a plate, weakened by a system of N arbitrarily oriented cracks. In terms of the known functions $g_n'(x_n)$, we find the stress intensity coefficients [13] at the vertices of an arbitrary crack

$$k_{1n}^{\pm} - ik_{2n}^{\pm} = \mp \lim_{x_n\to\pm a_n}\left[\frac{\sqrt{a_n^2-x_n^2}}{\sqrt{a_n}}\,g_n'(x_n)\right] \qquad (n=1,\ 2,\ \ldots,\ N) \tag{3.1}$$

Here the upper sign corresponds to the right-hand vertex of the crack and the lower sign to the left-hand vertex. On the basis of the formulas (2.4), (2.5), (3.1), we write, with an accuracy of quantities $O(\lambda^5)$, the values of the stress intensity coefficients

$$k_{1n}^{\pm} - ik_{2n}^{\pm} = -\frac{1}{\pi\sqrt{a_n}}\int_{-a_n}^{a_n}\sqrt{\frac{a_n \pm t}{a_n \mp t}}\,p_n(t)\,dt \mp \lambda^2\sqrt{a_n}\sum_{k=1}^{N}{}'\frac{\varepsilon_{nk}^2}{4}H_0(\pm 1)\times$$
$$(a_{nk11}Q_{k0} + b_{nk11}\overline{Q}_{k0}) \mp \lambda^3\sqrt{a_n}\sum_{k=1}^{N}{}'\frac{\varepsilon_{nk}^2}{8}[H_0(\pm 1)(a_{nk22}Q_{k1} + b_{nk22}\overline{Q}_{k1})\varepsilon_{nk} +$$
$$\varepsilon_{kn}H_1(\pm 1)(a_{nk21}Q_{k0} + b_{nk21}\overline{Q}_{k0})] \mp$$
$$\lambda^4\sqrt{a_n}\sum_{k=1}^{N}{}'\frac{\varepsilon_{nk}^2}{4}\left\{-\frac{1}{2}H_0(\pm 1)\sum_{r=1}^{N}{}'\frac{\varepsilon_{rk}^2}{4}[a_{nk11}(a_{kr11}Q_{r0} + b_{kr11}\overline{Q}_{r0}) +\right.$$
$$b_{nk11}(\bar{a}_{kr11}\overline{Q}_{r0}\bar{b}_{kr11}Q_{r0})] + \frac{1}{4}\Big[H_2(\pm 1)\ \varepsilon_{kn}^2(a_{nk31}Q_{k0} + b_{nk31}\overline{Q}_{k0}) +$$
$$\varepsilon_{nk}\varepsilon_{kn}H_1(\pm 1)(a_{nk32}Q_{k1} + b_{nk32}\overline{Q}_{k1}) + H_0(\pm 1)\varepsilon_{nk}^2\left(a_{nk33}\frac{Q_{k0} + 2Q_{k2}}{2} +\right.$$
$$\left.\left.\left. b_{nk33}\frac{\overline{Q}_{k0} + 2\overline{Q}_{k2}}{2}\right)\right]\right\} + O(\lambda^5) \qquad (n = 1, 2, \ldots, N). \tag{3.2}$$

Where

$$Q_{ks} = \frac{1}{\pi a_k^{2+s}}\int_{-a_k}^{a_k} t^s p_k(t)\sqrt{a_k^2 - t^2}\,dt \tag{3.3}$$

The formulas (3.2) give the solution of the problem for an arbitrary load (1.1). However, if concentrated forces are applied to the boundaries of the cracks at the points $x_k = \xi_k$, i.e. when $p_k(x_k) = (P_k - iQ_k)\,\delta(x_k - \xi_k)$ (where $\delta(x)$ is the delta function), then in the formulas (3.2) we have

$$Q_{ks} = \frac{P_k - iQ_k}{\pi a_k^{2+s}}\xi_k^s\sqrt{a_k^2 - \xi_k^2}$$

For the uniformly distributed load

$$p_k(x_k) = -(p_k - iq_k) = -s_k \tag{3.4}$$

along the boundaries, we obtain from (3.3)

$$Q_{k0} = -\frac{s_k}{2}, \quad Q_{k1} = 0, \quad Q_{k2} = -\frac{s_k}{8}, \ldots$$

We note that from the relations (3.2) we can obtain the solution of the problem for a plate weakened by a periodic system of cracks. In particular, setting in the formulas (2.2), (2.3), (3.2), (3.4), $a_k = a$, $\alpha_k = \alpha$, $d_{nk}e^{i\beta nk} = (n-k)\,d\,g_k(x_k) = g(x_k)$, $s_k = s$ and letting N tend to infinity, we obtain in the case of a constant load on the cracks

$$k_1^{\pm} - ik_2^{\pm} = \sqrt{a}\left\{s + \frac{\pi^2\lambda^2}{24}[s\cos 2\alpha + \bar{s}(e^{-2i\alpha} - e^{-4i\alpha})] + \frac{\pi^4\lambda^4}{128}\left\{s\left[\frac{1}{5}\cos 4\alpha +\right.\right.\right.$$
$$\left.\left.\left.\frac{2}{9}\cos^2 2\alpha + \frac{4}{9}(1 - \cos 2\alpha)\right] + 2\bar{s}(e^{-4i\alpha} - e^{-6i\alpha})\left(\frac{1}{5} + \frac{2}{9}e^{2i\alpha}\cos 2\alpha\right)\right\}\right\} + O(\lambda^6) \tag{3.5}$$

The conditions of limiting equilibrium around any of the vertices of the cracks are written in the form [12]

$$f_n(k_{1n}^{+}, k_{2n}^{+}) = \frac{1}{\sqrt{2}}\cos^3\frac{\gamma_n^{\pm}}{2}\left[k_{1n}^{+} - 3k_{2n}^{+}\operatorname{tg}\frac{\gamma_n^{\pm}}{2}\right] = \frac{K}{\pi} \tag{3.6}$$

Here K is the characteristic of the material strength in the propagation of the crack and $\gamma_n^{\pm}$ are the initial angles of the crack propagation.

In the general case, all the vertices of the cracks are under different conditions. We will consider that the limiting equilibrium state appears in the plate as soon as fracture starts at least at one of the vertices, i.e. the following condition holds

$$\min\{f_n(k_{1n}^{\pm}, k_{2n}^{\pm})\} = \frac{K}{\pi} \qquad (n = 1, 2, \ldots, N) \tag{3.7}$$

4. As an example we consider the limiting equilibrium of an infinite plate with two equal, arbitrarily oriented cracks in the case of a biaxial extension at infinity, i.e. for $N_x^{\infty} = q$, $N_y^{\infty} = p$. Setting in the formulas (3.2), (3.3), (2.3)

$$N = 2,\ a_1 = a_2 = a,\ \beta_{21} = \beta_{12} + \pi = \beta$$

$$p_k(x_k) = -\frac{p}{2}[1 + \eta + (1 - \eta)e^{-2i\alpha_k}], \qquad \eta = \frac{q}{p}$$

we find the stress intensity coefficients, and in terms of them, the critical value of the applied load for which fracture starts at any of the four vertices of the cracks

$$p_{*n}^{\pm} = \frac{K\sqrt{2}}{\pi\sqrt{a}\cos^3\gamma_n^{\pm}/2}\left(l_{1n}^{\pm} - 3l_{2n}^{\pm}\operatorname{tg}\frac{\gamma_n^{\pm}}{2}\right)^{-1}$$

$$l_{1n}^{\pm} - il_{2n}^{\pm} = \frac{1}{p\sqrt{a}}(k_{1n}^{\pm} - ik_{2n}^{\pm}) \qquad (n = 1, 2)$$

The magnitude of the fracture load $p_*(\alpha_1, \alpha_2, \beta, \lambda, \eta)$ for a plate with cracks of a given orientation is deter-

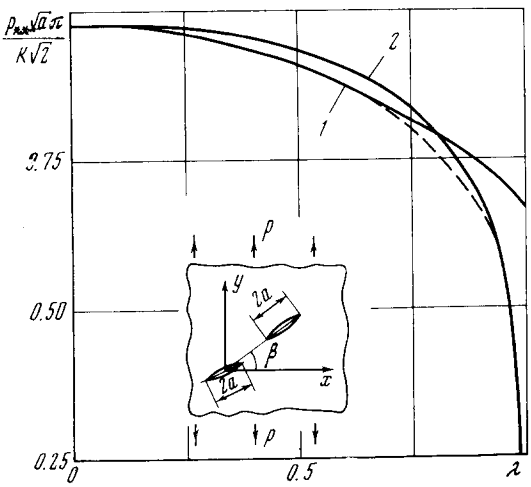

Fig. 2

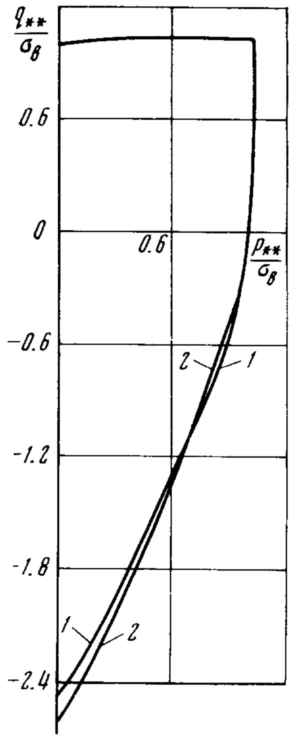

Fig. 3

mined by the condition (3.7).

In Fig. 2 we give the diagram of the dependence of the quantity

$$p_{**}(\lambda, \eta) = \min p_*(\alpha_1, \alpha_2, \beta, \lambda, \eta), \quad 0 \leqslant \alpha_1, \alpha_2, \beta < 2\pi \tag{4.1}$$

on λ (curve *1*) in the case of a monoaxial extension of the plate at infinity ($\eta = 0$). The curve *2* characterizes the similar dependence for a plate with two colinear cracks, obtained on the basis of the exact solution [14]. For the values $\lambda \to 1$, the above constructed solution does not hold; obviously, in this case the minimal limiting load $p_{**} \to 0$. Therefore one can expect that for $\lambda \geqslant 0{,}7$ the dependence of p_{**} on λ is interpolated by the curve given by the dashed line. From here it follows that the dependence of p_{**} on λ differs little (within the limits of 10%) from the corresponding dependence for colinear cracks. A similar conclusion holds also for the other values of λ.

In Fig. 3 we give the diagram of the minimum limiting stresses p_{**} (4.1), related to the average value of the technical strength of the material with defects of the given form [15] for $\lambda = 0{,}5$ (curve *1*). Curve *2* represents the behavior of a plate with an isolated crack of reduced length, i.e. the length for which the minimum limiting loads for a uniaxial extension of the plate with one and with two cracks are equal.

The diagrams of the limiting stresses for plates weakened by colinear cracks and by one crack of reduced length coincide [14], while the dependence of $p_{**}(\lambda, \eta)$ on the parameter λ for small values of η differs little from the corresponding dependence in the case of colinear cracks. Therefore we can conclude that the diagram of the limiting stresses, taking into account the interaction of differently oriented cracks, varies little qualitatively, not only in the case when the cracks are situated at large distances, but also when they are closer to each other.

BIBLIOGRAPHY

1. Ivlev, D. D., On the theory of quasi-brittle fracture. PMTF, №6, 1967.
2. Paris, P. C. and Sih, G. C. M., The analysis of the state of stress around cracks. Applied Problems in Fracture Viscosity. Moscow, "Mir", 1968.
3. Savin, G. N. and Panasiuk, V. V., The development of investigations in the theory of the limiting equilibrium of brittle bodies with cracks. Prikl. Mekhan. Vol. 4, №1, 1968.
4. Koiter, W. T., An infinite row of parallel cracks in an infinite elastic sheet. In: Problems of Continuum Mechanics. Philadelphia, SIAM, 1961.
5. Echina, N. M. and Pal'tsun, N. V., The elastic equilibrium of a plate, weakened by a system of cracks. Prikl. Mekhan., Vol. 7, №7, 1971.
6. Parton, V. Z., On one estimate of the mutual strengthening of cracks having a chessboard distribution. PMTF, №5, 1965.
7. Isida, M., Analysis of stress intensity factors for plates containing a random array of cracks. Bull. JSME, Vol. 13, №59, 1970.
8. Vitvitskii, P. M. and Leonov, M. Ia., Problems of the mechanics of a real solid body. №1, 1962.
9. Libatskii, L. L., The application of singular integral equations for the determination of the critical forces in plates with cracks. Fiz.-Khim. Mekhan. Materialov, Vol. 1, №4, 1965.
10. Muskhelishvili, N. I., Some Basic Problems of the Mathematical Theory of Elasticity. Moscow, "Nauka", 1966.
11. Gakhov, F. D., Boundary Value Problems. Moscow, Fizmatgiz, 1963.

12. Panasiuk, V. V., The Limiting Equilibrium of Brittle Bodies with Cracks. Kiev, "Naukova Dumka", 1968.
13. Irvin, G. R., Handbuch der Physik. Bd. 6, Berlin, Springer-Verlag, 1958.
14. Berezhnitskii, L. T., Panasiuk, V. V. and Arone, A. G., On the problem of the interactions between cracks situated along a single straight line. Fiz.-Khim. Mekhan. Materialov, Vol. 7, №2, 1971.
15. Libatskii, L. L. and Panasiuk, V. V., On the construction of the diagram of the limiting equilibrium of brittle bodies with sharp defects. PMTF, №3, 1970.

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STRESSES IN A SYMMETRICALLY LAMINATED PLATE WEAKENED BY A CENTRAL CRACK

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The axisymmetric state of stress of a piecewise-homogeneous infinite plate bonded from parallel layers and weakened by a transverse slit (crack) is considered. This problem is interesting in connection with some questions of computing the strength of rock strata. An investigation of the problem reduces to the solution of an integral equation in a function characterizing the change in the slit shape. The singularity of the solution is isolated, permitting a detailed study of the stress field including the edges and ends of the slit. Some numerical results are presented.

1. Let us consider the state of stress of a plate rigidly bonded together from strips of different elastic characteristics. The layers are assumed elastic, isotropic, and symmetric relative to the middle layer in both the elastic and geometric characteristics. The middle of the strip is slit perpendicularly to the boundary, and the plate itself is subjected to tension along the layers (Fig. 1). Let us take the following boundary conditions on the contour of the slit:

$$\sigma_x = p(y), \quad \tau_{xy} = 0, \quad x = 0, \quad |y| < 1 \tag{1.1}$$

where $p(y)$ is an even function.

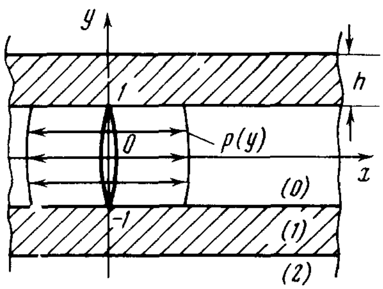

Fig. 1

The quantities referring to the middle layer (0), the layers (1) and the semi-infinite plates (2) will be denoted by the indices 0, 1, 2, respectively. Taking account of more general boundary conditions, such as addition of layers between the medium (1) and (2), is not difficult in principle and the form taken for the problem